7 Impact

7.1 Introduction

The previous chapters have dealt almost exclusively with *static* loading. We turn now to the more commonly encountered case of *dynamic* loading. Dynamic loading includes both *impact*, the subject of this chapter, and *fatigue*, which will be introduced in Chapter 8.

Impact loading is also called *shock, sudden*, or *impulsive* loading. The reader has inevitably experienced and observed many examples of impact loading—driving a nail or stake with a hammer, breaking up concrete with an air hammer, automobile collisions (even minor ones such as bumper impacts during careless parking), dropping of cartons by freight handlers, razing of buildings with an impact ball, automobile wheels dropping into potholes, and so on.

Impact loads may be divided into three categories in order of increasing severity: (1) rapidly moving loads of essentially constant magnitude, as produced by a vehicle crossing a bridge, (2) suddenly applied loads, such as those in an explosion, or from combustion in an engine cylinder, and (3) direct-impact loads, as produced by a pile driver, drop forge, or vehicle crash. These are illustrated schematically in Figure 7.1. In Figure 7.1*a*, mass m is held so that it just touches the top of spring k and is suddenly released. Dashpot c (also called a damper or shock absorber) adds a frictional supporting force that prevents the full gravitational force mg from being applied to the spring immediately. In Figure 7.1*b* there is no dashpot, so the release of mass m results in an instantaneous application of the full force mg. In Figure 7.1*c*, not only is the force applied instantaneously, but the mass acquires kinetic energy before it strikes the spring.

The significant thing about the dashpot action in Figure 7.1*a* is that it results in a *gradual* application of the load mg. If the load is applied slowly enough, it can be

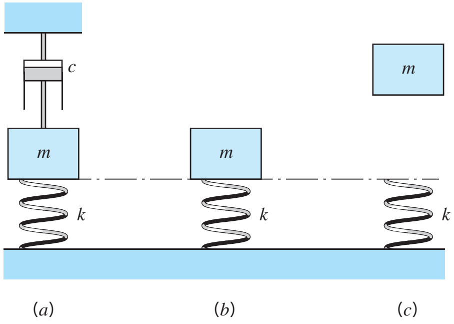

Figure 7.1
Three levels of impact loading produced upon instantaneous release of mass m.

considered as static. *The usual way of distinguishing between impact and static loading in this situation is to compare the time required for applying the load with the natural period of vibration of the undamped mass on the spring.*

[For the reader not yet acquainted with elementary vibration theory, imagine that the mass in Figure 7.1*b* is attached to the spring, that it is pushed down and then suddenly released. The mass will then vibrate up and down, with a fixed interval between consecutive times that it is in the "full up" or "full down" position. This time interval is the *natural period of vibration* of the mass on the spring. The relationship between this period (τ, s), the mass (m, kg or lb $\cdot$ s^2/in.), and the spring constant (k, N/m or lb/in.) is

$$\tau = 2\pi\sqrt{\frac{m}{k}} \qquad \textbf{(a)}$$

Thus, the *greater* the mass and the *softer* the spring the *longer* the period of vibration (or, the lower the natural frequency of vibration).]

If the time required to apply the load (i.e., to increase it from zero to its full value) is greater than three times the natural period, dynamic effects are negligible and static loading may be assumed. If the time of loading is less than half the natural period, there is definitely an impact. Of course, there is a "gray area" in between—see Table 7.1.

Impact loads can be compressive, tensile, bending, torsional, or a combination of these. The sudden application of a clutch and the striking of an obstruction by the bit of an electric drill are examples of torsional impact.

An important difference between static and impact loading is that statically loaded parts must be designed to *carry loads*, whereas parts subjected to impact must be designed to *absorb energy.*

Material strength properties usually vary with speed of load application. In general, this works out favorably because both the yield and ultimate strengths tend to increase with speed of loading. (Remember, though, that rapid loading tends to promote brittle fracture, as noted in Section 6.2.) Figure 7.2 shows the effect of strain rate on tensile properties of mild steel.

One of the problems in applying a theoretical analysis of impact to actual engineering problems is that often the time rates of load application and of strain development can only be approximated. This sometimes leads to the use of empirically determined stress impact factors, together with the static strength properties of the material. This practice works out well when good empirical data are available that apply closely to the part being designed. An example is the use of a stress impact factor of 4 in designing automotive suspension parts. Even when the use of these empirical factors is justified, it is important for the engineer to have a good understanding of the basic fundamentals of impact loading.

TABLE 7.1 Type of Loading

Load Type	**Time Required to Apply Load (s)**
Static loading	$t_{\text{applied loading}} > 3\tau$
"Gray area"	$\frac{1}{2}\tau < t_{\text{applied loading}} < 3\tau$
Dynamic loading	$t_{\text{applied loading}} < \frac{1}{2}\tau$

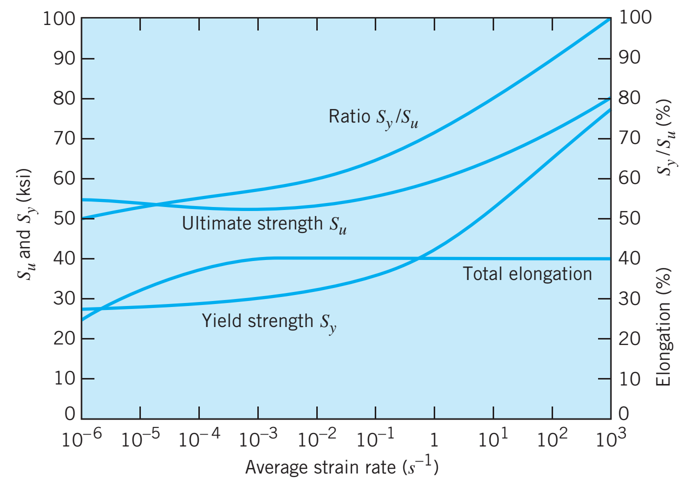

FIGURE 7.2
Effect of strain rate on tensile properties of mild steel at room temperature [2].

7.2 Stress and Deflection Caused by Linear and Bending Impact

Figure 7.3 shows an idealized version of a freely falling mass (of weight W) impacting a structure. (The structure is represented by a spring, which is appropriate because *all* structures have *some* elasticity.) To derive from Figure 7.3 the simplified equations for stress and deflection, the same assumptions are made as when deriving the equation for the natural frequency of a simple spring–mass system: (1) the mass of the structure (spring) is negligible, (2) deflections within the mass itself are negligible, and (3) damping is negligible. These assumptions have some important implications.

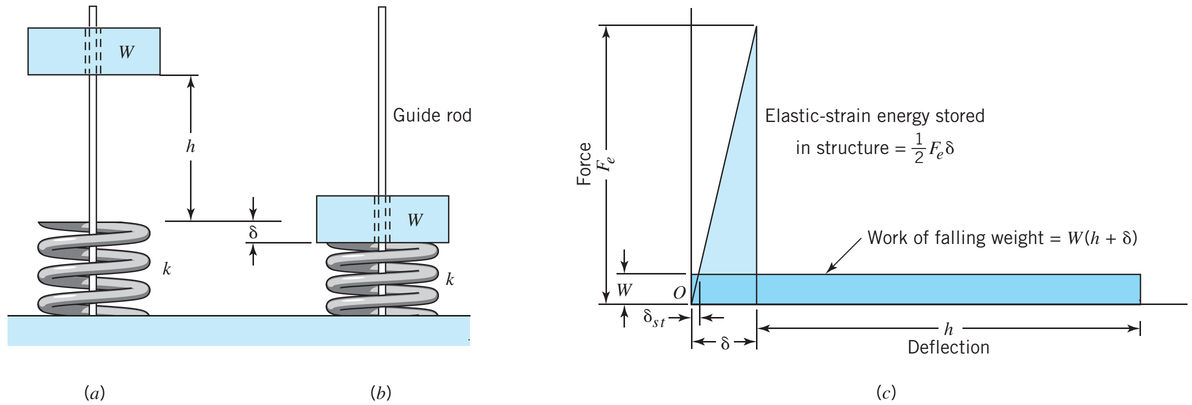

FIGURE 7.3
Impact load applied to elastic structure by falling weight: (*a*) initial position; (*b*) position at instant of maximum deflection; (*c*) force–deflection-energy relationships.

1. The first assumption implies that the dynamic deflection curve (i.e., the instantaneous deflections resulting from impact) is identical to that caused by the static application of the same load, multiplied by an *impact factor.* In reality, the dynamic deflection curve inevitably involves points of higher *local* strain (hence, higher local stress) than does the static curve.
2. *Some* deflection must inevitably occur within the impacting mass itself. To the extent that it does, a portion of the energy is absorbed within the mass, thereby causing the stresses and deflections in the structure to be a little *lower* than the calculated values.
3. Any actual case involves some (though perhaps very little) friction damping in the form of windage, rubbing of the mass on the guide rod and end of the spring (in Figure 7.3), and internal friction within the body of the deflecting structure. This damping can cause the actual stresses and deflections to be significantly less than those calculated from the idealized case.

Keeping the above limitations in mind, the following analysis of the idealized case provides an understanding of basic impact phenomena, together with equations that are very helpful as a *guide* in dealing with linear impact.

In Figure 7.3, the falling mass is such that (in the gravitational field involved) it has a weight, W (newtons or pounds). The structure is assumed to respond to the impact elastically, with a spring constant of k (newtons per meter or pounds per inch). The maximum value of deflection that is due to impact is δ (meters or inches). F_e is defined as an *equivalent static force* that would produce the same deflection δ; that is, $F_e = k\delta$. The static deflection that exists after the energy is damped out and the weight comes to rest on the structure is designated by δ_{st}, where $\delta_{st} = W/k$.

Equating the potential energy given up by the falling mass with the elastic energy absorbed by the spring (structure),

$$W(h + \delta) = \tfrac{1}{2} F_e \, \delta \qquad \textbf{(b)}$$

Note that the factor of $\frac{1}{2}$ appears because the spring takes on the load *gradually.*

By definition, since $F_e = k\delta$ and $k = W/\delta_{st}$

$$F_e = (\delta/\delta_{st})\, W \quad \text{or} \quad \delta/\delta_{st} = F_e/W \qquad \textbf{(c)}$$

Substituting Eq. c into Eq. b gives

$$W(h + \delta) = \frac{1}{2}\frac{\delta^2}{\delta_{st}} W \qquad \textbf{(d)}$$

Equation d is a quadratic equation in δ, which is solved routinely to give

$$\delta = \delta_{st}\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right) \qquad \textbf{(7.1)}$$

Substitution of Eq. c in Eq. 7.1 gives

$$F_e = W\left(1 + \sqrt{1 + \frac{2h}{\delta_{st}}}\right) \tag{7.2}$$

Since the structure (spring) is assumed to respond elastically to the impact, the stress produced is proportional to the load. The term in parentheses in Eqs. 7.1 and 7.2 is called the *impact factor.* It is the factor by which the load, stress, and deflection caused by the dynamically applied weight, W, exceed those caused by a slow, static application of the same weight.

In some cases it is more convenient to express Eqs. 7.1 and 7.2 in terms of velocity at impact v (meters per second or inches per second) instead of height of fall h. For free fall, the relationship between these quantities is

$$v^2 = 2gh \quad \text{or} \quad h = \frac{v^2}{2g} \tag{e}$$

where g is the acceleration of gravity measured in meters per second per second or inches per second per second.

Substitution of Eq. e in Eqs. 7.1 and 7.2 gives

$$\delta = \delta_{st}\left(1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}}\right) \tag{7.1a}$$

and

$$F_e = W\left(1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}}\right) \tag{7.2a}$$

Reducing distance h to zero with v equal to zero gives the special case of a *suddenly applied load*, for which the impact factor—in Eqs. 7.1 and 7.2—is equal to 2. This may have been one basis for designers in the past sometimes doubling safety factors when impact was expected.

In many problems involving impact, the deflection is almost insignificant in comparison to h (see Figure 7.3). For this case, where $h \gg \delta_{st}$, Eqs. 7.1 and 7.2 can be simplified to

$$\delta = \delta_{st}\sqrt{\frac{2h}{\delta_{st}}} = \sqrt{2h\delta_{st}} \tag{7.3}$$

$$F_e = W\sqrt{\frac{2h}{\delta_{st}}} = \sqrt{2Whk} \tag{7.4}$$

Similarly, Eqs. 7.1a and 7.2a simplify to

$$\delta = \delta_{st}\sqrt{\frac{v^2}{g\delta_{st}}} = \sqrt{\frac{\delta_{st}v^2}{g}} \tag{7.3a}$$

$$F_e = W\sqrt{\frac{v^2}{g\delta_{st}}} = \sqrt{\frac{v^2kW}{g}} \tag{7.4a}$$

In the preceding four equations, gravity was considered *only* as the means for developing the velocity of the weight at the point of impact (the further action of gravity after impact being neglected). Hence, Eqs. 7.3a and 7.4a apply also to the case of a *horizontally* moving weight striking a structure, where the impact velocity v is developed by means other than gravity. In this case, δ_{st} is the static deflection that *would* exist *if* the entire system were rotated 90° to allow the weight to act vertically upon the structure. Thus, regardless of the actual orientation,

$$\delta_{st} = W/k \qquad \textbf{(f)}$$

It is useful to express the equations for deflection and equivalent static force as functions of the impact kinetic energy U, where, from elementary physics,

$$U = \tfrac{1}{2}mv^2 = Wv^2/2g \qquad \textbf{(g)}$$

Substitution of Eqs. f and g into Eqs. 7.3a and 7.4a gives

$$\delta = \sqrt{\frac{2U}{k}} \qquad \textbf{(7.3b)}$$

$$F_e = \sqrt{2Uk} \qquad \textbf{(7.4b)}$$

Thus, the greater the energy, U, and the stiffer the spring, the greater the equivalent static force.

7.2.1 Linear Impact of Straight Bar in Tension or Compression

An important special case of linear impact is that of a straight rod or bar impacted in compression or in tension. The tensile case is illustrated schematically in Figure 7.4*a*. The tensile rod sometimes takes the form of a bolt. *If* the impact load is applied concentrically, and *if* stress concentration can be neglected (mighty big "ifs" usually!), then we can substitute into Eq. 7.4b the elementary expressions

$$\sigma = F_e/A \qquad \textbf{(h)}$$

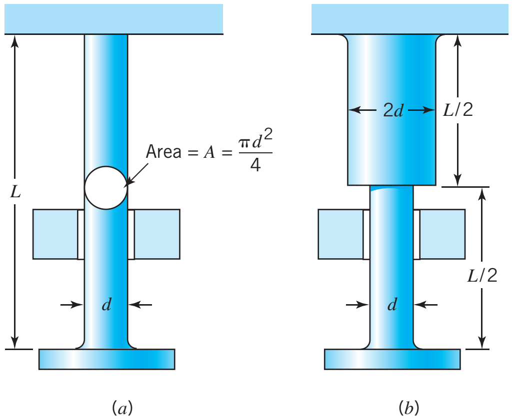

FIGURE 7.4
Tensile impact.

and

$$k = AE/L \tag{i}$$

where A and L are the rod cross-sectional area and length, respectively. The resulting equation is

$$\sigma = \sqrt{\frac{2UE}{AL}} = \sqrt{\frac{2UE}{V}} \tag{7.5}$$

where V is the volume of material in the rod.

Note the important implication of Eq. 7.5—the stress developed in the rod is a function of its *volume* irrespective of whether this volume is made up of a long rod of small area or a short rod of large area.

Solving Eq. 7.5 for U gives

$$U = \frac{\sigma^2 V}{2E} \tag{7.5a}$$

This shows the *impact energy capacity* of a straight rod to be a remarkably simple function of its volume, its modulus of elasticity, and the *square* of the allowable stress.

Despite the importance of this basic relationship, it should be emphasized that Eqs. 7.5 and 7.5a may, in practice, give results that are considerably optimistic—that is, give a calculated stress *lower* than the actual peak stress, and, correspondingly, a calculated energy capacity *greater* than that which actually exists. The main reasons for this are: (1) the stresses are not uniform throughout the member, due to stress concentration and nonuniformity of loading on the impacted surface, and (2) the impacted member has mass. The inertia resulting from the rod mass causes the impacted end of the rod to have a greater *local* deflection (hence, stress) than it would if inertial effects did not prevent the instantaneous distribution of deflection throughout the length of the rod. The effect of stress raisers is considered in Section 7.4. The quantitative effect of the mass of the struck member is left for more advanced works; see [1], [6], and [8].

7.2.2 Sample Problems for Linear and Bending Impact

SAMPLE PROBLEM 7.1 Axial Impact—Importance of Section Uniformity

Figure 7.4 shows two round rods subjected to tensile impact. How do their elastic energy-absorbing capacities compare? (Neglect stress concentration and use S_y as an approximation of the elastic limit.)

SOLUTION

Known: Two round rods of given geometry are subjected to tensile impact.

Find: Compare the elastic energy-absorbing capacities of the two rods.

Schematic and Given Data: See Figure 7.4.

Assumptions:

1. The mass of each rod is negligible.
2. Deflections within each impacting mass itself are negligible.

3. Frictional damping is negligible.
4. Each rod responds to the impact elastically.
5. The impact load is applied concentrically.
6. Stress concentration can be neglected.

Analysis:

1. The elastic capacity for Figure 7.4*a* is determined directly from Eq. 7.5a, where $\sigma = S_y$:

$$U_a = \frac{S_y^2 V}{2E}$$

2. In Figure 7.4*b*, the energy absorbed by the upper and lower halves must be determined separately. The smaller lower half is critical; it can be brought to a stress of S_y, and its volume is $V/2$ (where V = volume of the full-length rod in Figure 7.4*a*). Thus, energy capacity of the lower half is

$$U_{bl} = \frac{S_y^2 V/2}{2E} = \tfrac{1}{2} U_a$$

3. The same force is transmitted through the full length of the rod. The upper half has four times the area of the lower half; hence, it has four times the volume and only $\frac{1}{4}$ the stress. Thus the energy capacity of the upper half is

$$U_{bu} = \frac{(S_y/4)^2(2V)}{2E} = \tfrac{1}{8} U_a$$

4. The total energy capacity is the sum of U_{bl} and U_{bu}, which is *five-eighths the energy capacity of the constant diameter rod of Figure 7.4a*. Since the rod in Figure 7.4*b* has $2\frac{1}{2}$ times the volume and weight of the straight rod, it follows that the *energy capacity per pound of* the uniform-section rod is *four times as great* as that of the stepped rod.

Comment: The stress concentration in the middle of the stepped bar would further reduce its capacity and would tend to promote brittle fracture. This point is treated further in the next section.

SAMPLE PROBLEM 7.2 Relative Energy Absorption Capacity of Various Materials

Figure 7.5 shows a falling weight that impacts on a block of material serving as a bumper. Estimate the relative elastic-energy-absorption capacities of the following bumper materials.

Material	Density (kN/m^3)	Elastic Modulus (E)	Elastic Limit (S_e, MPa)
Soft steel	77	207 GPa	207
Hard steel	77	207 GPa	828
Rubber	9.2	1.034 MPa	2.07

Solution

Known: A weight falls on energy-absorbing bumpers of specified materials.

Find: Compare the elastic-impact capacity of the bumper materials.

Schematic and Given Data:

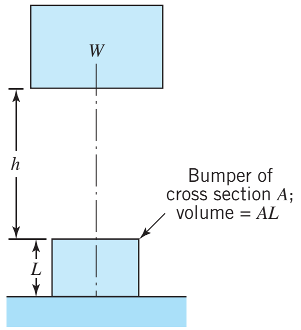

Figure 7.5
Impact loading of compression bumper.

Assumptions:

1. The mass of the bumper is negligible.
2. Deflections within the impacting weight itself are negligible.
3. Damping is negligible.
4. The bumper responds elastically.
5. The impact load is applied uniformly.

Analysis:

1. From Figure 7.3, the elastic strain energy absorbed is $\frac{1}{2}F_e\delta$, or the area under the force-deflection curve. At the elastic limit, $F_e = S_eA$, and $\delta = F_eL/AE$. Substitution of these values gives

$$U = \tfrac{1}{2}F_e\delta = \frac{S_e^2AL}{2E} = \frac{S_e^2V}{2E}$$

which, not surprisingly, corresponds exactly with Eq. 7.5a.

2. Substitution of the given material properties in the above equation indicates that on the basis of unit volume, the relative elastic-energy-absorption capacities of the soft steel, hard steel, and rubber are 1 : 16 : 20. On a unit mass or weight basis the relative capacities are 1 : 16 : 168.

Comment: The capacity per unit volume of a material to absorb elastic energy is equal to the area under the elastic portion of the stress–strain diagram and is called the *modulus of resilience* (R_m) of the material. The *total* energy absorption capacity in tension per unit volume of the material is equal to the total area under the stress–strain curve (extending out to fracture) and is sometimes called the *modulus of toughness* (T_m) of the material. In the above problem the two steels differed markedly in their moduli of resilience, but their relative toughnesses would likely be comparable.

SAMPLE PROBLEM 7.3 Bending Impact—Effect of Compound Springs

Figure 7.6 shows a wood beam supported on two springs and loaded in bending impact. Estimate the maximum stress and deflection in the beam, based on the assumption that the masses of the beam and spring can be neglected.

SOLUTION

Known: A 100-lb weight falls from a specified height onto a wood beam of known material and specified geometry that is supported by two springs.

Find: Determine the maximum beam stress and deflection.

Schematic and Given Data:

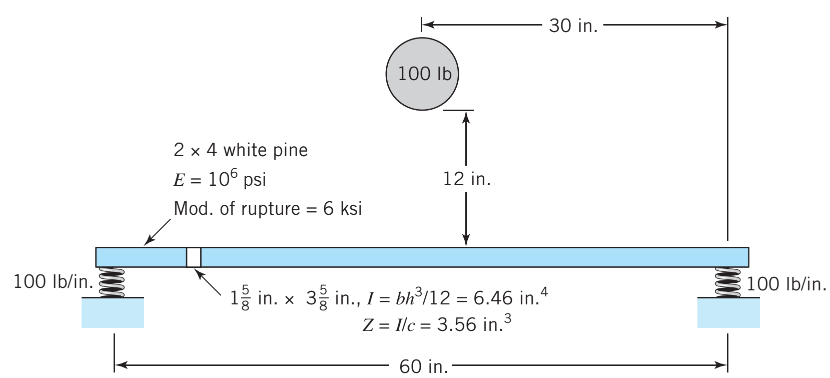

FIGURE 7.6
Bending impact, with compound spring.

Assumptions:

1. As stated in the problem, the masses of the beam and spring can be neglected.
2. The beam and springs respond elastically.
3. The impact load is applied uniformly at the center of the beam.

Analysis:

1. The static deflection for the beam only, supporting springs only, and total system are

$$\delta_{st}(\text{beam}) = \frac{PL^3}{48EI} = \frac{100(60)^3}{48(10^6)(6.46)} = 0.070 \text{ in.}$$

$$\delta_{st}(\text{springs}) = \frac{P}{2k} = \frac{100}{2(100)} = 0.50 \text{ in.}$$

$$\delta_{st}(\text{total}) = 0.070 + 0.50 = 0.57 \text{ in.}$$

2. From Eq. 7.1 or 7.2 the *impact factor* is

$$1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{24}{0.57}} = 7.6$$

3. Hence, the total impact deflection is $0.57 \times 7.6 = 4.3$ in., but the deflection of the beam itself is only $0.07 \times 7.6 = 0.53$ in.
4. The extreme-fiber beam stress is estimated from $F_e = 100 \times 7.6 = 760$ lb:

$$\sigma = \frac{M}{Z} = \frac{F_e L}{4Z} = \frac{760(60)}{4(3.56)} = 3200 \text{ psi}$$

Comments:

1. The estimated stress is well within the given *modulus of rupture* of 6000 psi. (The modulus of rupture is the computed value of *M/Z* at failure in a standard static test.)
2. If the supporting springs are removed, the total static deflection is reduced to 0.07 in., and the impact factor increases to 19.6. This would give a computed maximum beam stress of 8250 psi, which is greater than the modulus of rupture. If the inertial effect of the beam mass does not cause the actual stress to be very much higher than 8250 psi, it is possible that the "dynamic-strengthening effect" shown in Figure 7.2 would be sufficient to prevent failure. Because this effect is usually appreciable for woods, the results of standard beam impact tests are often included in references giving properties of woods.

7.3 Stress and Deflection Caused by Torsional Impact

The analysis of the preceding section could be repeated for the case of torsional systems, and a corresponding set of equations developed. Instead, advantage will be taken of the direct analogy between linear and torsional systems to write the final equations directly. The analogous quantities involved are

Linear	Torsional
δ, deflection (m or in.)	θ, deflection (rad)
F_e, equivalent static force (N or lb)	T_e, equivalent static torque (N · m or lb · in.)
m, mass (kg or lb · s^2/in.)	I, moment of inertia (N · s^2 · m or lb · s^2 · in.)
k, spring rate (N/m or lb/in.)	K, spring rate (N · m/rad or lb · in./rad)
v, impact velocity (m/s or in./s)	ω, impact velocity (rad/s)
U, kinetic energy (N · m or in. · lb)	U, kinetic energy (N · m or in. · lb)

The two following equations have the letter *t* added to the equation number to designate torsion:

$$\theta = \sqrt{\frac{2U}{K}} \qquad \textbf{(7.3bt)}$$

$$T_e = \sqrt{2UK} \qquad \textbf{(7.4bt)}$$

For the important special case of torsional impact of a solid round bar of diameter *d*:

1. From Table 5.1,

$$K = \frac{T}{\theta} = \frac{K'G}{L} = \frac{\pi d^4 G}{32L} \tag{i}$$

2. From Eq. 4.4 with T replaced by T_e,

$$\tau = \frac{16T_e}{\pi d^3} \tag{j}$$

3. Volume, $V = \pi d^2 L/4$ **(k)**

Substitution of Eqs. i, j, and k into Eq. 7.4bt gives

$$\tau = 2\sqrt{\frac{UG}{V}} \tag{7.6}$$

SAMPLE PROBLEM 7.4 Torsional Impact

Figure 7.7*a* shows the shaft assembly of a grinder, with an abrasive wheel at each end and a belt-driven sheave at the center. (The sheave can also be thought of as the armature of an electric motor.) When turning at 2400 rpm, the smaller abrasive wheel is accidentally jammed, causing it to stop "instantly." Estimate the resulting maximum torsional stress and deflection of the shaft. Consider the abrasive wheels as solid disks of density $\rho = 2000$ kg/m^3. The shaft is steel ($G = 79$ GPa), and its weight may be neglected.

SOLUTION

Known: The smaller wheel of a grinder turning at 2400 rpm is stopped instantly.

Find: Determine the maximum shaft stress and torsional deflection.

Schematic and Given Data:

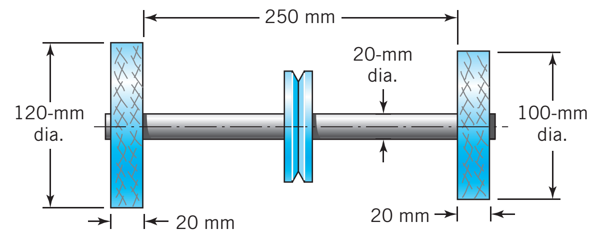

FIGURE 7.7*a*
Torsional impact of grinder shaft.

Assumptions:

1. The weight of the shaft and pulley may be neglected.
2. The shaft acts as a torsional spring and responds elastically to the impact.
3. Deflections within the abrasive wheels are negligible.

Analysis:

1. It is the energy in the 120-mm wheel that must be absorbed by the shaft. From the torsional equivalent of Eq. g, this is

$$U = \tfrac{1}{2} I\omega^2$$

where

$$I = \tfrac{1}{2} m r_{\text{wheel}}^2$$

and

$$m = \pi r_{\text{wheel}}^2 t\rho$$

2. Combining the preceding equations, we have

$$U = \tfrac{1}{4}\pi r_{\text{wheel}}^4 t\rho\omega^2$$

3. Substituting numerical values (with units of meters, kilograms, and seconds) gives

$$U = \tfrac{1}{4}\pi(0.060)^4(0.020)(2000)\left(\frac{2400 \times 2\pi}{60}\right)^2$$

$$U = 25.72\frac{\text{kg}\cdot\text{m}^2}{s^2} = 25.72\ \text{N}\cdot\text{m}$$

4. From Eq. 7.6

$$\tau = 2\sqrt{\frac{UG}{V}}$$

$$= 2\sqrt{\frac{(25.72)(79 \times 10^9)}{\pi(0.010)^2(0.250)}} = 321.7 \times 10^6\ \text{Pa}$$

or

$$\tau = 322\ \text{MPa}$$

5. The torsional deflection,

$$\theta = \frac{TL}{JG}$$

where $T = \tau J/r$ (i.e., $\tau = Tr/J$); hence,

$$\theta = \frac{\tau L}{rG} = \frac{(321.7 \times 10^6)(0.250)}{(0.010)(79 \times 10^9)} = 0.10\ \text{rad} = 5.7°$$

Comments:

1. The preceding calculations assumed that the stresses are within the elastic range. Note that no provision was made for stress concentration or for any superimposed bending load that would also be present as a result of the jamming. Torque applied to the sheave by the belt was also neglected, but this could be negligible because of belt slippage. In addition, it is only because of assumed belt slippage that the inertia of the driving motor is not a factor.
2. The effect of the shaft radius, r, on shaft shear stress, τ, and torsional deflection, θ, can be explored by computing and plotting the shaft stress and torsional deflection for a shaft radius from 5 mm to 15 mm, and for a shear modulus, G, of steel (79 GPa), cast iron (41 GPa) and aluminum (27 GPa)—see Figure 7.7*b*.

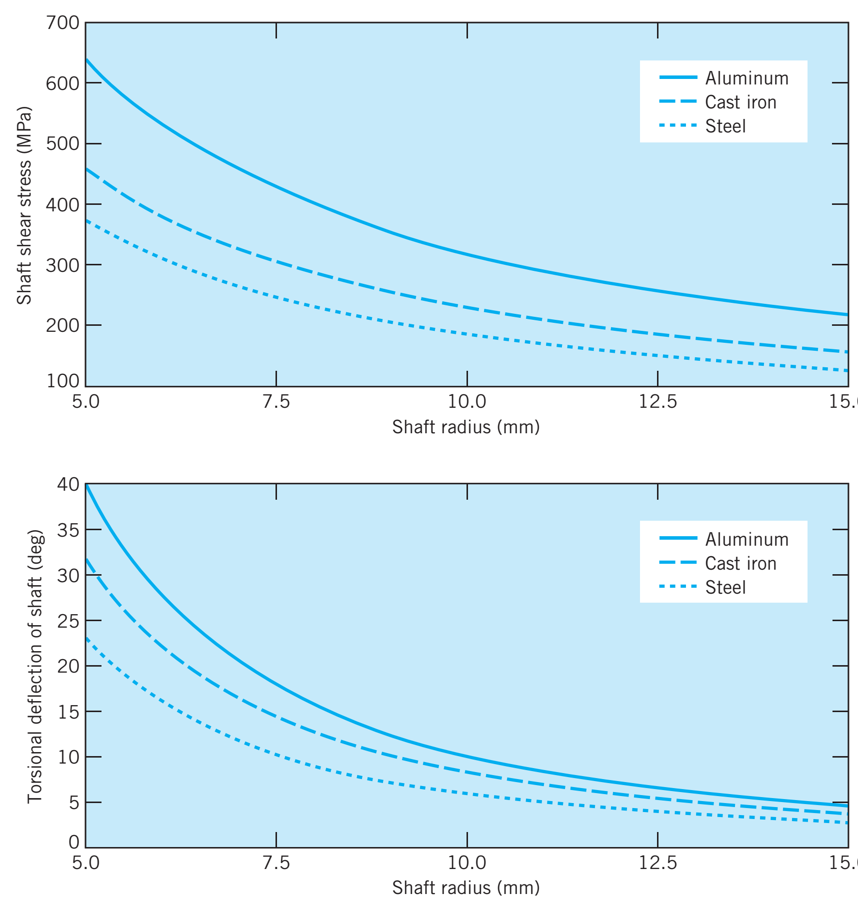

FIGURE 7.7*b*
Shear stress and torsional deflection vs. shaft radius.

3. For a 2024-T4 aluminum alloy shaft with a 10 mm radius, with $S_y = 296$ MPa (Appendix C-2) and with $S_{sy} = 0.58S_y = 172$ MPa, the shear stress is $\tau = 188$ MPa and the shaft rotation is $\theta = .174$ rad $= 10°$. Inspection of the plot of shaft shear stress versus shaft radius shows that shaft radius should be greater than 11 mm to avoid yielding in a 2024-T4 aluminum alloy shaft.

7.4 Effect of Stress Raisers on Impact Strength

Figure 7.8 shows the same tensile impact bar as Figure 7.4*a*, except that recognition is given to the fact that stress concentration exists at the ends of the bar. As with static loading it is *possible* that local yielding would redistribute the stresses so as to virtually nullify the effect of the stress raiser. But under impact loading the *time* available for plastic action is likely to be so short that brittle fracture (with an effective stress concentration factor almost as high as the theoretical value, K_t, obtained from a chart similar to Figure 4.35*b*) will sometimes occur even in a material that exhibits ductile behavior in the tensile test. In terms of the discussion in Section 6.2, adding a stress raiser and applying an impact load are both factors tending to raise

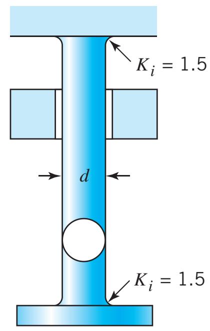

FIGURE 7.8
Plain impact bar.

the *transition temperature*—that is, cause brittle fracture without dropping to as low a temperature.

Because of the difficulty in predicting impact notch effects from theoretical considerations, standard notched impact tests are used, such as the *Charpy* and *Izod.* These, too, have their limitations, for notched impact strength varies markedly with size, shape, and nature of impact. Because of this, special laboratory tests that more closely simulate actual conditions are sometimes used.

SAMPLE PROBLEM 7.5 Notched Tensile Impact

Suppose that from special tests it has been determined that the *effective stress concentration factor for impact loading*, K_i, at the ends of the rod in Figure 7.8 is 1.5, as shown. How much does the stress raiser decrease the energy-absorbing capacity of the rod, as estimated from Eq. 7.5a?

SOLUTION

Known: A round rod subject to impact loading has a specified stress concentration at each end.

Find: Determine the effect of a stress raiser on rod energy-absorbing capacity.

Schematic and Given Data: See Figure 7.8.

Assumption: Under impact loading, the rod material exhibits brittle behavior.

Analysis: First, two observations: (a) If the rod is sufficiently long, the volume of material in the region of the end fillets is a very small fraction of the total and (b) the material at the critical fillet location cannot be stressed in excess of the material strength S. This means that *nearly all* the material can be considered as stressed to a uniform level that cannot exceed S/K_i, or, in this instance, $S/1.5$. Thus, a good approximation is that after considering the stress raiser, the same volume of material is involved, but at a stress level reduced by a factor of 1.5. Since the stress is squared in Eq. 7.5a, taking the notch into consideration *reduces the energy capacity by a factor of* 1.5^2*, or 2.25.*

SAMPLE PROBLEM 7.6 Notched Tensile Impact

Figure 7.9 shows the same impact bar as Figure 7.8, except that a sharp groove, with $K_i = 3$, has been added. Compare the impact-energy capacities of the bars in Figures 7.8 and 7.9.

SOLUTION

Known: A grooved impact bar and a plain impact bar are each subjected to impact loading.

Find: Compare the impact-energy capabilities of both bars.

Schematic and Given Data:

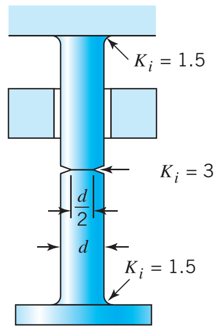

FIGURE 7.9
Grooved impact bar.

Assumption: The rod materials exhibit brittle behavior.

Analysis: In Figure 7.9, the impact capacity is limited to the value that brings the stress at the groove to the material strength S. Since the effective stress concentration factor is 3, the nominal stress level in the section of the groove is $S/3$. Because of the 4 : 1 area ratio, the nominal stress in the bulk of material (*not* in the groove plane) is only $S/12$. For a long bar, the percentage of volume near the groove is very small. Thus, with reference to Eq. 7.5a, the only substantial difference made by introducing the groove is to reduce the value of σ from $S/1.5$ to $S/12$. Since σ is squared in the equation, the groove reduces the energy capacity by a factor of 64; that is, the grooved bar has *less than 2 percent* of the energy-absorbing capacity of the ungrooved bar!

From this discussion, it follows that the effective design of an efficient energy-absorbing member comprises two key steps.

1. Minimize stress concentration as much as possible. (Always try to reduce the stress at the point where it is highest.)
2. Having done this, remove all possible "excess material" so that the stress everywhere is as close as possible to the stress at the most critical point. Removing this excess material does not reduce the *load* that the member can carry, and the *deflection* is increased. Since energy absorbed is the integral of force times deflection, energy-absorbing capacity is thereby increased. (Recall the dramatic example of this principle in Sample Problem 7.1.)

SAMPLE PROBLEM 7.7 Modifying a Bolt Design for Greater Impact Strength

Figure 7.10*a* shows a bolt that is subjected to tensile impact loading. Suggest a modified design that would have greater energy-absorbing capacity. How much increase in capacity would the modified design provide?

SOLUTION

Known: A standard bolt of specified geometry is to be modified for tensile impact loading.

Find: Modify the bolt geometry and estimate the increase in energy-absorbing capacity.

Schematic and Given Data:

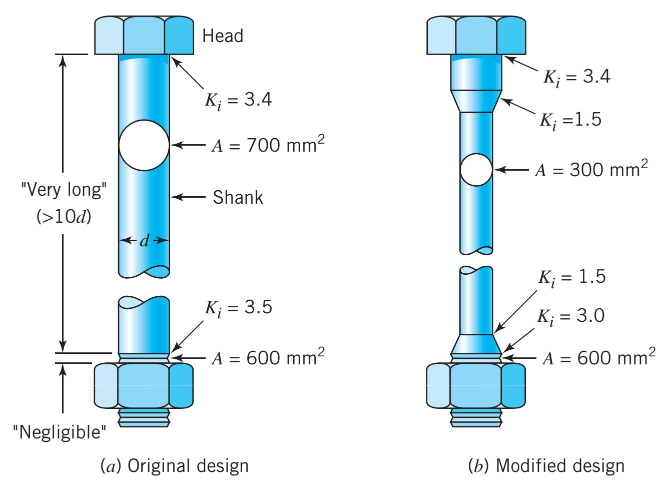

FIGURE 7.10
Bolt subjected to tensile impact.

Decisions: The following decisions are made in the design analysis.

1. Minimize stress concentration by using a thread with a smooth, generous fillet at the root.
2. Leave a short length of full-diameter shank under the bolt head to serve to center the bolt in the bolt hole.
3. Design for uniform stress throughout the bolt by reducing the diameter in the lesser-stressed portion of the shank.

Assumptions: Material of strength S is used for both bolts. Other assumptions are made as required throughout the design analysis.

Design Analysis:

1. Reduce stress concentration where it is most critical. The highest stress is at the thread (K_i = 3.5, acting at an area of only 600 mm^2). Assume that by modifying

the thread slightly to provide a smooth, generous fillet at the root, K_i can be reduced to 3.0, as shown in Figure 7.10*b*. The other point of stress concentration is in the fillet under the bolt head. This fillet must be small in order to provide adequate flat area for contact. Actually, there is no incentive to reduce stress concentration at that point because the stress there will be less than at the thread root, even with the modified thread design.[1]

$$\sigma = \frac{P}{A}K_i; \left(\frac{P}{700} \times 3.4\right)_{\text{fillet}} < \left(\frac{P}{600} \times 3.0\right)_{\text{thread root}}$$

2. Leave a short length of full-diameter shank under the bolt head to serve as a pilot to center the bolt in the bolt hole. The diameter in the rest of the shank can be reduced to make the shank stress nearly equal to the stress at the thread root. Figure 7.10*b* shows a reduced shank diameter that is flared out to the full diameter with a large radius, to give minimal stress concentration. On the basis of a conservative stress concentration estimate of 1.5, the shank area can be reduced to half the effective-stress area at the thread:

$$A = 600 \times \frac{1.5}{3.0} = 300 \text{ mm}^2$$

3. Assume that the bolt is sufficiently long so that the volume of uniformly stressed material in the central portion of the shank is the only volume that need be considered, and that the volumes in the two critical regions are proportional to the areas 700 and 300 mm^2. From Eq. 7.5a, $U = \sigma^2 V/2E$. Since E is a constant, the ratio of energy capacities for Figures 7.10*b* and 7.10*a* is

$$\frac{U_b}{U_a} = \frac{\sigma_b^2 V_b}{\sigma_a^2 V_a} \tag{m}$$

In Figure 7.10*a* the stress in the large volume of material in the shank is less than the material strength S because of *both* the stress concentration *and* the difference in area between the thread and shank. Thus, if $\sigma = S$ at the thread root, the shank stress is

$$\sigma_a = \frac{S}{3.5}\left(\frac{600}{700}\right) = 0.245S$$

Let the shank volume in Figure 7.10*a* be designated as V. In Figure 7.10*b*, the stress at the thread can again be S. Corresponding shank stress is

$$\sigma_b = \frac{S}{3.0}\left(\frac{600}{300}\right) = 0.667S$$

The shank volume in Figure 7.10*b* is V (300/700), or $0.429V$. Substituting these values in Eq. m gives

$$\frac{U_b}{U_a} = \frac{(0.667S)^2(0.429V)}{(0.245S)^2(V)} = 3.18$$

[1] This is usually, but not *always*, the case.

Comments:

1. The redesign has over three times the capacity of the original, as well as being lighter.
2. For a given volume of material, the bolt design with the more nearly uniform stress throughout will have the greater energy-absorbing capacity.

Two other designs of bolts with increased energy-absorbing capacity are illustrated in Figure 7.11. In Figure 7.11*a*, the full-diameter piloting surface has been moved to the center of the shank in order to provide alignment of the two clamped members. Figure 7.11*b* shows a more costly method of removing excess shank material, but it preserves nearly all the original torsional and bending strength of the bolt. The torsional strength is often important, for it influences how much the nut can be tightened without "twisting off" the bolt.

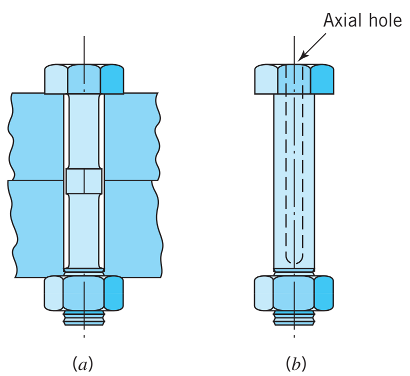

FIGURE 7.11
Bolts designed for energy absorption.

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Problems

Section 7.2

7.1 According to marketing materials from a major athletic shoe manufacturer,

the human foot is punished by the forces of impact it encounters during athletic activities. This makes cushioning essential to the development of a good athletic shoe. A cushioning system has been designed to help protect the foot from these harmful forces by helping to absorb impact. This cushioning system has the ability to absorb shock by dissipating vertical impact and dispersing it into a horizontal plane. The cushioning system is available for absorbing shock when it is placed in the forefoot and/or rear foot of the shoes mid-sole.

Using the force flow concept, explain, if possible, how a gel cushioning system could absorb shock by dissipating vertical impact and dispersing it into a horizontal plane.

7.2 The previous chapters have dealt essentially with considerations of stress, strain, and strength arising from *static* loading. The present chapter deals with impact, and the subsequent chapter treats fatigue—both are cases of dynamic loading. Impact loading is also referred to as *shock*, *sudden*, or *impulsive* loading. Impact loads may be torsional and/or linear in nature. How does *impact* loading differ from *static* loading?

7.3 A tensile impact bar, similar to the one in Figure 7.4*a*, fractured in service. Because the failure happened to occur near the center, a naive technician makes a new bar exactly like the old one except that the middle third is enlarged to twice the diameter of the ends. Assuming that stress concentration can be neglected (not very realistic), how do the impact capacities of the new and old bars compare?

7.4 A vertical member is subjected to an axial impact by a 100-lb weight dropped from a height of 2 ft (similar to Figure 7.4*a*). The member is made of steel, with $S_y = 45$ ksi, $E = 30 \times 10^6$ psi. Neglect the effect of member mass and stress concentration. What must be the length of the member in order to avoid yielding if it has a diameter of (a) 1 in., (b) $1\frac{1}{2}$ in., (c) 1 in. for half of its length and $1\frac{1}{2}$ in. for the other half?
[Ans. 90.5 in., 40 in., 125.2 in.]

7.5 A car skidded off an icy road and became stuck in deep snow at the road shoulder. Another car, of 1400-kg mass, attempted to jerk the stuck vehicle back onto the road using a 5-m steel tow cable of stiffness $k = 5000$ N/mm. The traction available to the rescue car prevented it from exerting any significant force on the cable. With the aid of a push from bystanders, the rescue car was able to back against the stuck car and then go forward and reach a speed of 4 km/h at the instant the cable became taut. If the cable is attached rigidly to the center of mass of each car, estimate the maximum impact force that can be developed in the cable, and the resulting cable elongation (see Figure P7.5).

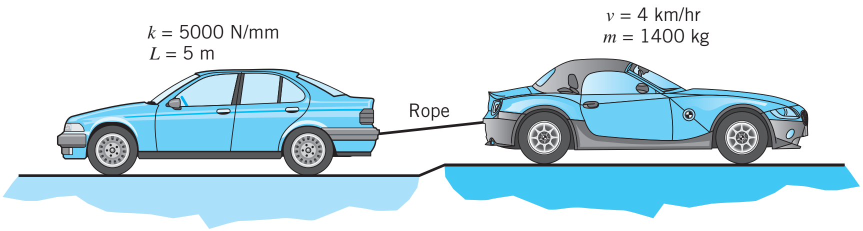

FIGURE P7.5

7.6 Repeat Problem 7.5 where the rescue car has a 2800-kg mass.

7.7 Repeat Problem 7.5 where the steel tow cable has a stiffness of 2500 N/mm.

7.8 Repeat Problem 7.5 where the rescue car reached a speed of 8 km/hr at the instant the cable became taut.

7.9 The rescue attempt in Problem 7.5 resulted in only slight movement of the stuck car because the cable force decayed so quickly to zero. Besides, concern was felt about possible damage to the car attachment points because of the high "instantaneous" force developed. One witness to the proceedings brought a 12-m elastic cable of overall stiffness only 2.4 N/mm and suggested that it be tried. Because of the longer length of the elastic cable, its use enabled the rescue car to reach 12 km/h at the point of becoming taut. Estimate the impact force developed and the resulting cable elongation. If the stuck vehicle does not move significantly until the rescue car has just come to a stop, how much energy is stored in the cable? (Think of this in terms of the height from which a 100-kg mass would have to be dropped to represent an equivalent amount of energy, and consider the potential hazard if the cable should break or come loose from either car.) What warnings would you suggest be provided with elastic cables sold for this purpose?

7.10 A tow truck weighing 6000 lb attempts to jerk a wrecked vehicle back onto the roadway using a 15-ft length of steel cable 1 in. in diameter ($E = 12 \times 10^6$ psi for the cable). The truck acquires a speed of 3 mph at the instant the cable slack is taken up, but the wrecked car does not move. (a) Estimate the impact force applied to the wrecked vehicle and the stress produced in the cable. (b) The cable breaks in the middle, and the two 7.5-ft halves are connected in parallel for a second try. Estimate the impact force and cable stress produced if the wrecked vehicle still remains fixed.

[Ans. (a) 60.6 ksi, 47,600 lb]

7.11 Repeat Sample Problem 7.3, except use a 1.0×1.0-in. ($b \times h$) aluminum beam.

7.12 A 5-ton elevator is supported by a standard steel cable of 2.5-in.^{2} cross section and an effective modulus of elasticity of 12×10^6 psi. As the elevator is descending at a constant 400 ft/min, an accident causes the top of the cable, 70 ft above the elevator, to stop suddenly. Estimate the maximum elongation and maximum tensile stress developed in the cable (see Figure P7.12).

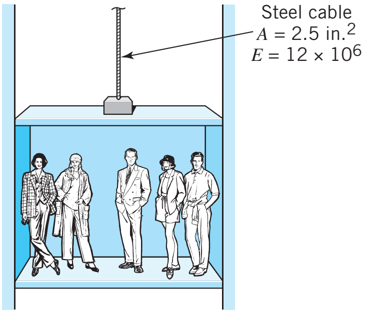

FIGURE P7.12

7.13 For the 5-ton elevator described in Problem 7.12, explore the effect of cable length on the maximum elongation and maximum tensile stress developed in the cable by computing and plotting the cable elongation and tensile stress for a cable length above the elevator of 1 to 500 feet.

7.14 A 60-foot-long, 950-lb gin pole used to raise sections of a communication tower is suspended by a standard steel cable of 0.110-in.^{2} cross section with an effective modulus of elasticity of 12×10^6 psi. As the gin pole descends at a constant speed of 30 ft/min, an accident causes the top of the cable, 70 ft above the gin pole, to stop suddenly. Estimate the maximum elongation and maximum tensile stress developed in the cable.

Section 7.3

7.15 The vertical drive shaft in Figure P2.31 is 20 mm in diameter, 650 mm long, and made of steel. The motor to which it is attached at the top is equivalent to a steel flywheel 300 mm in diameter and 25 mm thick. When the vertical shaft is rotating at 3000 rpm, the propeller strikes a heavy obstruction, bringing it to a virtually instantaneous stop. Assume that the short horizontal propeller shaft and the bevel gears have negligible flexibility. Calculate the elastic torsional shear stress in the vertical shaft. (Since this stress far exceeds any possible torsional elastic strength, a shear pin or slipping clutch would be used to protect the shaft and associated costly parts.)

7.16 Repeat Problem 7.15 where the vertical shaft is rotating at 6000 rpm.

7.17 Repeat Problem 7.15 where the vertical shaft is 10 mm in diameter.

7.18 Repeat Problem 7.15 where the vertical drive shaft is 325 mm long.

7.19 Figure P7.19 shows a cantilevered steel rod with a 90º bend lying in a horizontal plane. Weight W is dropped onto the end of the rod from height h. If the steel has a yield strength of 50 ksi, what combinations of W and h are required to produce yielding of the rod? Neglect the weight of the rod and neglect transverse shear stresses. Assume the maximum-distortion-energy theory of failure applies.

[Ans. $Wh \geq 92.4$ in.-lb, based on an additional simplifying assumption.]

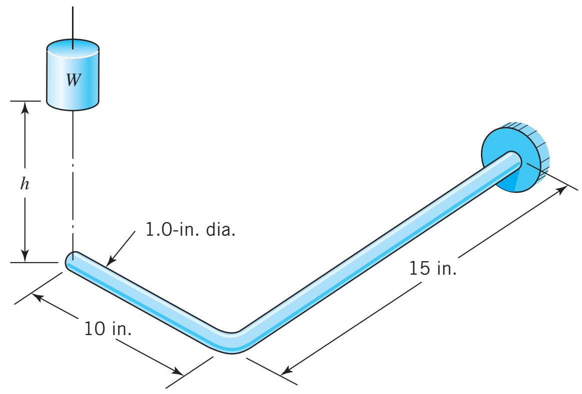

FIGURE P7.19

Section 7.4

7.20 For the tensile impact bar shown in Figure P7.20, estimate the ratio of impact energy that can be absorbed with and without the notch (which reduces the diameter to 24 mm) Assume that $K = K_i = K_t$.

[Ans.: 0.06 : 1]

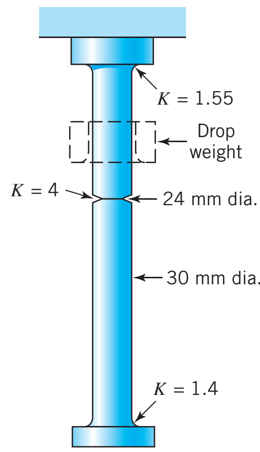

FIGURE P7.20

7.21 A platform is suspended by long steel rods as shown in Figure P7.21*a*. Because heavy items are sometimes dropped on the platform, it is decided to modify the rods as shown in Figure P7.21*b* to obtain greater energy-absorbing capacity. The new design features enlarged ends, blended into the main portion with generous fillets, and special threads giving less stress concentration. (Assume that $K = K_i = K_t$).

(a) What is the smallest effective threaded section area A (Figure P7.21*b*) that would provide maximum energy-absorbing capability?

(b) Using this value of A (or that of the next larger standard thread size), what increase in energy-absorbing capacity would be provided by the new design?

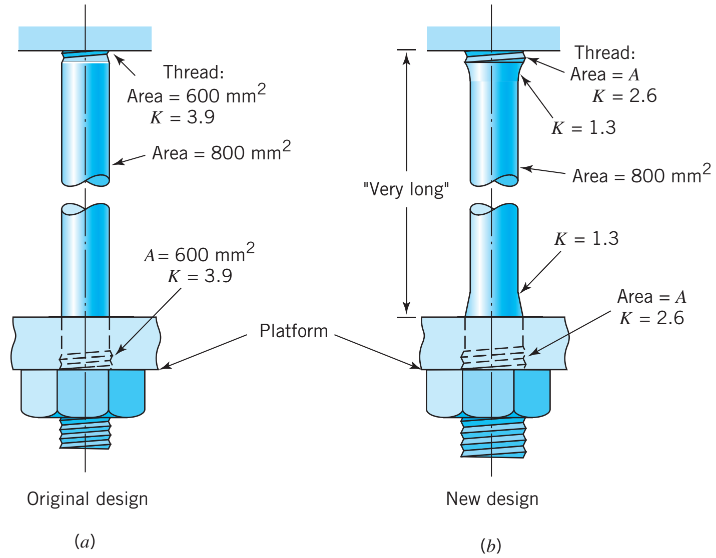

FIGURE P7.21

7.22 The initial design of a bolt loaded in tensile impact is shown in Figure P7.22*a*. The bolt fractures next to the nut, as shown. A proposed redesign, Figure P7.22*b* involves drilling an axial hole in the unthreaded portion and incorporating a larger fillet radius under the bolt head.

(a) What is the theoretically optimum diameter of the drilled hole?

(b) Using this hole size, by what approximate factor do the modifications increase the energy-absorbing capacity of the bolt?

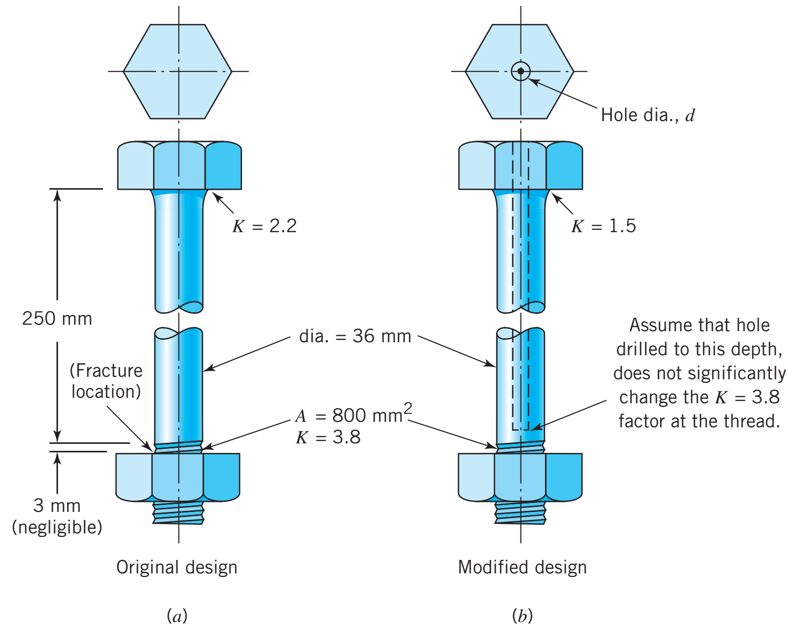

FIGURE P7.22

7.23 Figure P7.23 shows a tensile impact bar with a small transverse hole. By what factor does the hole reduce the impact-energy-absorbing capacity of the bar?

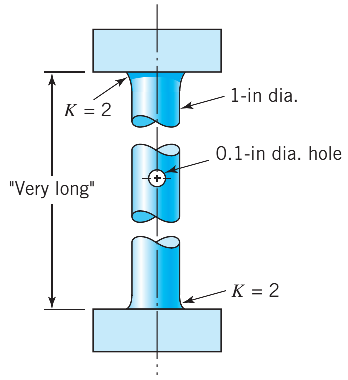

FIGURE P7.23

7.24D Redesign the bolt loaded in tensile impact and shown in Figure P7.22*a* to *increase* the energy absorbing capacity by a factor of 3 or more.

7.25D Redesign the plain impact bar shown in Figure 7.8 of the text to *reduce* the impact-energy-absorbing capacity of the impact bar by a factor of 2 or more. Assume that the bar has a diameter, $d = 1.0$ in.